Admissibilization of Continuous Descriptor Systems

M. Chaabane[1], O. Bachelier[2], R. A. Ramírez-Mendoza[3], and D. Mehdi[2]

[1] Automatic Control Research Unit, Electrical Engineering Department, Sfax National Engineering School, B.P. 805 Route Menzel Chaker km 0.5, 3038 Sfax, Tunisia
mohamed.chaabane@ipeis.rnu.tn
[2] L.A.I.I, ESIP, 40 Avenue du Recteur Pineau, 86022 Poitiers Cedex, France,
{olivier.bachelier,mehdi.driss}@univ-poitiers.fr
[3] Tecnológico de Monterrey, Campus Monterrey, Avenida Eugenio Garza Sada 2501 Sur, Monterrey, N.L., México
ricardo.ramirez@itesm.mx

Abstract. This paper deals with the problems of robust admissibility and admissibilization for uncertain continuous descriptor systems. We propose a new necessary and sufficient condition in term of a strict Linear Matrix Inequality (LMI) for a nominal continuous descriptor system to be admissible (stable, regular and impulse free). Based on this, the state feedback admissibility problem is solved and the solution extended to the case of uncertain descriptor systems. Finally numerical examples are given to illustrate the results.
Key words : Continuous descriptor systems, Admissibility, Lyapunov equation, Feedback control,

1 Introduction

Intuitively, singular state space description of linear systems is more general than conventional state space description. In particular, a descriptor form includes information about static as well as dynamic constraints. Singular system, both continuous and discrete, has been of interest in the literature since they have many applications (see, [4]), for instance in electrical circuits network, robotic and economics. It is fair to say that descriptor models give a more complete class of dynamical models than the conventional state space systems.
Many classical concepts and results obtained for conventional systems have been extended to descriptor systems. Let us quote for instance controllability and observability, pole assignment, stability analysis [8,6]) and stabilization techniques as well as results including robustness aspects [9,7,13].
The natural *Generalized Lyapunov Equation* (GLE) [8] was proven in [6] to fail unless the system is in its Weirstrass form and the authors of [6] proposed a new GLE equivalent to that given in [11]. In [9], the authors modified the GLE given

© A. Gelbukh, S. Suárez. (Eds.)
Advances in Computer Science and Engineering.
Research in Computing Science 23, 2006, pp. 229-239

Received 07/07/06
Accepted 07/10/06
Final version 13/10/06

in [11] and proposed an equivalent matrix inequality condition.
In the available literature on descriptor systems, there are two kinds of stabilization problems for singular continuous-time systems. One consists in designing a state feedback controller in such a way that the closed-loop system is regular, impulse-free and stable or equivalently admissible. The other is to design a state feedback controller in order to make the closed-loop system regular and stable. In many approaches, the system model is transformed into a special form and it is understandable that this way of doing is not very appropriate in the presence of uncertainty.
Concerning the stability analysis and the stabilization problem, a number of approaches assuming or not the regularity of the descriptor system have been proposed in the literature. Let us quote for instance [2,4,12] among those assuming the regularity and [4,12] among those not assuming the regularity.
Robust control of linear state space systems has been the focus of much attention during the past decades and various aspects and approach for analysis and control design for linear uncertain systems have been investigated, see for instance [15]. In the available literature we easily note that quadratic stability and stabilization approaches have taken a lion's share. The quadratic stability or stabilization is characterized by a determination of a unique so-called Lyapunov matrix which gives the approach an inherent conservatism. Many results have been reported in quadratic stability analysis and/or stabilization (see for instance [1,15,14] and the reference therein).
Recently the Parameter Dependent Lyapunov (PDL) approach has been introduced to reduce the conservatism of the quadratic approach. The PDL approach consists in expressing the Lyapunov matrix as a function of the uncertainty and with the help of some slack additional variables the approach yields a significant reduction of conservatism [5,3].
In this paper, a Linear Matrix Inequality (LMI) formulation is adopted to express necessary and/or sufficient conditions for the admissibility of continuous time descriptor systems. The proposed approach can be understood as the LMI-correspondant formulation of the GLE proposed in [6]. It is known that strict inequality conditions are tractable and reliable especially with the available LMI software solver.
The state feedback stabilization problem is solved by means of the admissibility of the closed loop. When the system contains uncertainties, the present method is extended to solve the robust state feedback admissibility problem particularly through a PDL approach.
This paper is organized as follows. Section 2 gives the problem formulation and some preliminary definitions. Section 3 gives the result on admissibility for continuous time descriptor systems. In section 4, main result to solve the static feedback problem for the nominal descriptor systems is given, whereas Section 5 presents the result for uncertain singular systems. Section 6 presents illustrative examples. Section 7 concludes the paper.

2 Problem formulation

Consider the following continuous time descriptor system

$$E\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input. The matrix E may be singular, we shall assume that $\mathsf{rank}(E) = r \leq n$. A and B are known real constant matrices with appropriate dimensions.

Definition 1. *[4]*

1. *The pair (E, A) is said to be regular if $det(sE - A)$ is not identically zero.*
2. *The pair (E, A) is said to be impulse free if $deg(det(sE - A)) = \mathsf{rank}(E)$.*

In the rest of the paper the notation is standard unless it is otherwise specified. $L > 0$ ($L < 0$) means that the matrix L is a symmetric and positive definite matrix (a symmetric and negative definite). In the sequel, $\mathsf{Sym}\{\cdot\}$ is defined as $\mathsf{Sym}\{X\} = \left(X + X^\top\right)$ for any matrix X.

It is worth noting that the stability property for conventional systems is no more sufficient for singular systems but completed by the regularity and the absence of impulses and this lead us to introduce the notion of admissibility.

Definition 2. *[4] [7]*
The continuous time singular system (1) is said to be admissible if it is regular, impulse free and stable.

In order to characterize the admissibility of a singular system let us recall that for a pair (E, A) there exists a transformation couple (U, V) such as

$$\bar{E} = UEV = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad \bar{A} = UAV = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix}.$$

It comes then that the singular system is said to be admissible if

$$\text{matrix} \quad \bar{A}_{22} \quad \text{is non singular} \tag{2}$$

and there exits a symmetric and positive definite matrix $\bar{X}_{11}$ such that

$$\mathsf{Sym}\left\{((\bar{A}_{11} - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21})\bar{X}_{11})\right\} < 0. \tag{3}$$

Indeed, condition (2) means that the considered system is regular and impulse free whereas condition (3) states that matrix $(\bar{A}_{11} - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21})$ is stable. Notice that conditions (3–2) are not tractable for uncertain systems and it is preferable to use directly the system matrices. Both conditions will be combined in a unique condition as in [6]. The associated Lyapunov matrix will be neither symmetric nor positive definite. The positivity will be required only on a fraction of the Lyapunov matrix in order to satisfy condition (3).

To solve the admissibility problem, we propose a Lyaponuv-type admissibility condition, which is expressed by a strict LMI as given in Theorem 1. The goal of this paper is to find a static state feedback controller $u(t) = Kx(t)$ such that the closed loop system $(E, A + BK)$ is admissible. This is defined, in this paper, as the static feedback admissibility problem of descriptor systems. The solvability of the above problems will be characterized by some LMI conditions. If the derived LMI conditions are feasible, the feedback gain matrix can be obtained. If the system contains polytopic uncertainties, the results can be modified to find the static state feedback gain in such a way that the closed loop uncertain system is admissible.

3 Admissibility analysis

Consider the singular system described by the pair (E, A) and define $E^{\perp}$ and $E^{\dagger}$ as follows

$$E^{\perp} = V\,(I - UEV)\,U, \qquad E^{\dagger} = U^{\top}\,(I - UEV)\,U^{-\top}, \tag{4}$$

with U and V, two non singular matrices satisfying

$$UEV = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}.$$

Theorem 1. *The continuous time singular system (E, A) is admissible if and only if there exist some matrices $X = X^{\top}$, Y and G such that condition*

$$\begin{bmatrix} 0 & \left(XE^{\top} + E^{\perp}YE^{\dagger}\right)^{\top} \\ \left(XE^{\top} + E^{\perp}YE^{\dagger}\right) & 0 \end{bmatrix} + \mathsf{Sym}\left\{\begin{bmatrix} A \\ -I \end{bmatrix} G\,[U^{-\top}V^{\top}]\right\} < 0 \tag{5}$$

is feasible.

Proof of Theorem 1 :
By virtue of [10, Theorem 2.3.12] condition (5) is equivalent to

$$\begin{aligned} [I \quad A] \begin{bmatrix} 0 & \left(XE^{\top} + E^{\perp}YE^{\dagger}\right)^{\top} \\ \left(XE^{\top} + E^{\perp}YE^{\dagger}\right) & 0 \end{bmatrix} \begin{bmatrix} I \\ A^{\top} \end{bmatrix} & \\ = \mathsf{Sym}\left\{AXE^{\top} + AE^{\perp}YE^{\dagger}\right\} < 0 & \qquad (6) \\ [U \quad -V^{-1}] \begin{bmatrix} 0 & \left(XE^{\top} + E^{\perp}YE^{\dagger}\right)^{\top} \\ \left(XE^{\top} + E^{\perp}YE^{\dagger}\right) & 0 \end{bmatrix} \begin{bmatrix} U^{\top} \\ -V^{-\top} \end{bmatrix} & \\ = -\mathsf{Sym}\left\{\bar{X}\bar{E}^{\top} + \bar{E}^{\perp}\bar{Y}\bar{E}^{\dagger}\right\} < 0 & \qquad (7) \end{aligned}$$

with

$$\begin{aligned}\bar{X} &= V^{-1}XV^{-\top},\\ \bar{Y} &= UYU^{\top},\\ \bar{E} &= UEV,\\ \bar{E}^{\perp} &= V^{-1}E^{\perp}U^{-1} = I - UEV,\\ \bar{E}^{\dagger} &= U^{-\top}E^{\dagger}U^{\top} = I - UEV.\end{aligned}$$

Note that condition (7) implies that the 11-block of $\bar{X}$ is positive definite.

First we will show that matrix $\bar{A}_{22}$ is non singular or in other words the considered system is regular and impulse free. For this, multiplying both sides of (6) by U and $U^{\top}$ respectively, we get

$$\begin{aligned}&\mathsf{Sym}\left\{UAXE^{\top}U^{\top}\right\} + \mathsf{Sym}\left\{UAE^{\perp}YE^{\dagger}U^{\top}\right\}\\ &= \mathsf{Sym}\left\{\bar{A}\bar{X}\bar{E}^{\top}\right\} + \mathsf{Sym}\left\{\bar{A}\bar{E}^{\perp}\bar{Y}\bar{E}^{\dagger}\right\}\\ &= \mathsf{Sym}\left\{\begin{bmatrix} * & 0\\ * & 0\end{bmatrix}\right\} + \mathsf{Sym}\left\{\begin{bmatrix} 0 & *\\ 0 & (\bar{A}_{22}\bar{Y}_{22})\end{bmatrix}\right\} < 0\end{aligned}$$

which shows that we have necessarily

$$\mathsf{Sym}\left\{\bar{A}_{22}\bar{Y}_{22}\right\} < 0$$

and this means that $\bar{A}_{22}$ is invertible or in other words that the system is regular and impulse free. Note that the stars here corresponds to terms with no much relevance at this step.

To prove the stability of the singular system, we consider the two matrices

$$\Sigma = \begin{bmatrix} I & 0\\ -\bar{A}_{22}^{-1}\bar{A}_{21} & \bar{A}_{22}\end{bmatrix} \qquad \text{and} \qquad \Gamma = \begin{bmatrix} I & -\bar{A}_{12}\bar{A}_{22}^{-1}\\ 0 & I\end{bmatrix}$$

that transform the matrix $\bar{A}$ in a diagonal form as

$$\bar{\bar{A}} = \Gamma\bar{A}\Sigma = \begin{bmatrix}\bar{\bar{A}}_{11} & 0\\ 0 & I\end{bmatrix}$$

with

$$\bar{\bar{A}}_{11} = \bar{A}_{11} - \bar{A}_{12}\bar{A}_{22}^{-1}\bar{A}_{21}.$$

If we multiply both sides of (6) by ΓU and its transpose we get

$$\mathsf{Sym}\left\{\bar{\bar{A}}\bar{\bar{X}}\bar{\bar{E}}^{\top}\right\} + \mathsf{Sym}\left\{\bar{\bar{A}}\bar{\bar{E}}^{\perp}\bar{\bar{Y}}\bar{\bar{E}}^{\dagger}\right\} = \mathsf{Sym}\left\{\begin{bmatrix}\bar{\bar{A}}_{11}\bar{X}_{11} & 0\\ * & 0\end{bmatrix}\right\} + \mathsf{Sym}\left\{\begin{bmatrix} 0 & 0\\ * & *\end{bmatrix}\right\} < 0$$

with

$$
\begin{aligned}
&\bar{\bar{E}} = \Gamma \bar{E} \Sigma = \bar{E},\\
&\bar{\bar{X}} = \Sigma^{-1} \bar{X} \Sigma^{\top -1} = \begin{bmatrix} \bar{X}_{11} & * \\ * & * \end{bmatrix},\\
&\bar{\bar{E}}^{\perp} = \Sigma^{-1} \bar{E}^{\perp} \Gamma^{-1} = \bar{E}^{\perp},\\
&\bar{\bar{Y}} = \Gamma \bar{Y} \Gamma^{\top}.
\end{aligned}
$$

Then, it comes that we have

$$
\mathsf{Sym}\left\{\bar{\bar{A}}_{11}\bar{X}_{11}\right\} < 0 \tag{8}
$$

which means that the singular system is stable since matrix $\bar{X}_{11}$ is symmetric and positive definite according to condition (7) and this ends the proof of the theorem. $\nabla\nabla\nabla$

4 Admissibilization by state feedback

In this section we address the problem of admissibilisation by state feedback for the singular system given by

$$
E\dot{x}(t) = Ax(t) + Bu(t) \tag{1}
$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input.

The control law given by a state feedback is then

$$
u(t) = Kx(t) \tag{9}
$$

where the gain K, of appropriate dimension is computed in such a way that the singular closed loop system is admissible.

Theorem 2. *The continuous singular system is admissibilizable if and only if the* LMI *problem*

$$
\begin{bmatrix} 0 & \left(XE^{\top} + E^{\perp}YE^{\dagger}\right)^{\top} \\ \left(XE^{\top} + E^{\perp}YE^{\dagger}\right) & 0 \end{bmatrix} + \mathsf{Sym}\left\{\begin{bmatrix} AG + BR \\ -G \end{bmatrix}\left[U^{-\top}V^{\top}\right]\right\} < 0 \tag{10}
$$

is feasible in the variables X, Y, R and G. Moreover, the state feedback is given by

$$
K = RG^{-1}. \tag{11}
$$

Proof of Theorem 2

The closed loop singular system is admissible if and only if there exist two matrices X and Y and a state feedback K such that

$$\begin{bmatrix} 0 & \left(XE^\top + E^\perp Y E^\dagger\right)^\top \\ \left(XE^\top + E^\perp Y E^\dagger\right) & 0 \end{bmatrix} + \mathsf{Sym}\left\{\begin{bmatrix} (A+BK) \\ -I \end{bmatrix} G \left[\, U^{-\top} \; V^\top \,\right]\right\} < 0 \quad (12)$$

and it becomes clear that a change of variable $R = KG$ yields condition (10). This ends the proof by virtue of Theorem 1. $\nabla\nabla\nabla$

From Theorem 2 we can easily be aware that this approach has the advantage to be nicely applicable in the case the system is uncertain with a polytopic description of uncertainty which is the subject of the next section.

5 Robust Admissibilisation

In this section, we consider the feedback admissibilization problem for systems containing uncertainties. The descriptor system is characterized by the pair $(E,\ A(\bar{\alpha}), B(\bar{\alpha}))$ where matrices $A(\bar{\alpha})$ and $B(\bar{\alpha})$ belong to a polytope, that is

$$[\, A(\bar{\alpha}) \quad B(\bar{\alpha}) \,] = \sum_{i=1}^{p} \alpha_i \, [\, A_i \quad B_i \,]$$

with

$$\alpha_i \geq 0,\ i = 1,\ \ldots,\ p, \qquad \sum_{i=1}^{p} \alpha_i = 1 \qquad \text{and} \qquad \bar{\alpha} = [\, \alpha_1 \quad \ldots \quad \alpha_p \,]\,.$$

The problem for a system corrupted by uncertainty is to preserve its performances for all admissible uncertainties or in other terms for every instance of matrices $A(\bar{\alpha})$ and $B(\bar{\alpha})$.

Definition 3. *The uncertain singular system is robustly admissible if and only if it is admissible for every instance of the uncertainty $\bar{\alpha}$.*

Precisely, the continuous singular system will be robustly admissible if the 22-block of $UA(\bar{\alpha})V$ is invertible and all the finite poles are located in the open left half plane, that is, there exists a symmetric positive definite matrix $\bar{X}_{11}(\bar{\alpha})$ such that

$$\mathsf{Sym}\left\{((\bar{A}_{11}(\bar{\alpha}) - \bar{A}_{12}(\bar{\alpha})\bar{A}_{22}^{-1}(\bar{\alpha})\bar{A}_{21}(\bar{\alpha}))\bar{X}_{11})(\bar{\alpha})\right\} < 0.$$

In this case, the continuous singular system will be termed as robustly admissible.

Unfortunately, solving the previous equation is out of reach for the time being and instead one can thought of matrix $\bar{X}_{11})(\bar{\alpha})$ as a unique matrix over the uncertainty set, that is condition

$$\mathsf{Sym}\left\{((\bar{A}_{11}(\bar{\alpha}) - \bar{A}_{12}(\bar{\alpha})\bar{A}_{22}^{-1}(\bar{\alpha})\bar{A}_{21}(\bar{\alpha}))\bar{X}_{11})\right\} < 0$$

holds with of course the invertibility of matrix $\bar{A}_{22}$ then in this case the continuous singular system will be termed as quadratically admissible.

It is worth noting that quadratic admissibility implies robust admissibility but the converse is, in general, false. If quadratic admissibility is only sufficient for robust admissibility, it matters to find (sufficient) conditions for robust admissibility that are less conservative than quadratic admissibility. In other words, it is important to derive conditions implicitly involving matrices X and Y that are not constant but dependent on the uncertainty. One possibility is that matrices X and Y comply with

$$[\,X \quad Y\,] = \sum_{i=1}^{p} \alpha_i\,[\,X_i \quad Y_i\,], \tag{13}$$

where X_i and Y_i are valid to assess the admissibility of extreme models. Based upon these notions, the next theorem is proposed:

Theorem 3. *The uncertain closed loop singular system is robustly admissible if there exist matrices X_i, Y_i, for $i = 1, \ldots, p$, and two matrices G and R such as the* LMI *problem*

$$\begin{bmatrix} 0 & \left(X_iE^\top + E^\perp Y_iE^\dagger\right)^\top \\ \left(X_iE^\top + E^\perp Y_iE^\dagger\right) & 0 \end{bmatrix} + \mathsf{Sym}\left\{\begin{bmatrix} A_iG + B_iR \\ -G \end{bmatrix}[\,U^{-\top}\; V^\top\,]\right\} < 0 \tag{14}$$

is feasible. The admissible feedback gain is then given by

$$K = RG^{-1}.$$

Proof of Theorem 3 :

Let matrices X and Y be defined as in (13), then multiplying conditions (14) by α_i and summing up from 1 to p one gets the same conditions as in Theorem 2. $\nabla\nabla\nabla$

6 Illustrative example

Example 1. Consider a continuous time descriptor system as in (1) described by the parameters :

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}, \qquad A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & 4 & 4 \\ -1 & 2 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.0 & 0.2 \\ 1.0 & 0.2 \\ 0.9 & 0.8 \end{bmatrix}.$$

The finite poles are located at 0.3820 and 2.6180 which implies that the open loop is not admissible.

Taking

$$U = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \quad \text{and} \quad V = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

and applying Theorem 1 we obtain :

$$X = \begin{bmatrix} -1.3116 & -1.2550 & 1.3530 \\ -1.2550 & -1.1265 & 1.2720 \\ 1.3530 & 1.2720 & -1.3670 \end{bmatrix} \times 10^4, \; Y = \begin{bmatrix} 0.6075 & 2.5598 & 1.4604 \\ -0.3609 & 1.5548 & 0.6442 \\ 1.4604 & 0.6442 & -3.1074 \end{bmatrix} \times 10^4,$$

$$G = \begin{bmatrix} -3.8214 & 7.6429 & -3.8498 \\ -7.2060 & 4.3087 & 4.2920 \\ 3.8544 & -4.2934 & 0.0071 \end{bmatrix} \times 10^4, \; R = \begin{bmatrix} 1.3466 & 0.3268 & -2.0933 \\ -0.2569 & 0.4626 & -0.2203 \end{bmatrix} \times 10^5$$

and the state feedback gain :

$$K = RG^{-1} = \begin{bmatrix} 2.5061 & -2.6310 & 1.0596 \\ -1.9832 & -2.2806 & -6.8965 \end{bmatrix}$$

which renders the resulting closed loop system admissible with the finite closed loop poles located at -0.9965 and -0.2841 and the 22-block of UAV equaling -2.7512.

Example 2. Consider the uncertain singular system defined by

$$E = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

with matrices A and B belonging to a polytope whose vertices are given by

$$[\, A_1 \quad A_2 \,] = \left[\begin{array}{ccc|ccc} 1 & -1 & 0 & -1.0000 & 0.0680 & -0.8158 \\ 3 & 0 & 4 & -0.2500 & 0.6842 & 2.9136 \\ 1 & 4 & 6 & 1.7500 & 0.5482 & 4.5452 \end{array}\right],$$

$$[\, A_3 \quad A_4 \,] = \left[\begin{array}{ccc|ccc} 2.0000 & 0.0800 & 2.2162 & -2.0000 & 0.0672 & -1.8166 \\ 2.8900 & 0.7562 & 6.3456 & -1.4500 & 0.6834 & 1.4728 \\ -1.1100 & 0.5962 & 1.9132 & 2.5500 & 0.5490 & 5.1060 \end{array}\right].$$

The input matrices

$$[\, B_1 \, B_2 \, B_3 \, B_4 \,] = \left[\begin{array}{cc|cc|cc|cc} 0.0122 & 0.0422 & 0.0122 & 0.0402 & 0.0122 & 0.0422 & 0.0122 & 0.0402 \\ 0.3590 & 0.1652 & 0.3750 & 0.1632 & 0.3990 & 0.1652 & 0.3350 & 0.1632 \\ 0.5291 & 0.2949 & 0.5591 & 0.3289 & 0.5971 & 0.3349 & 0.4911 & 0.2889 \end{array}\right]$$

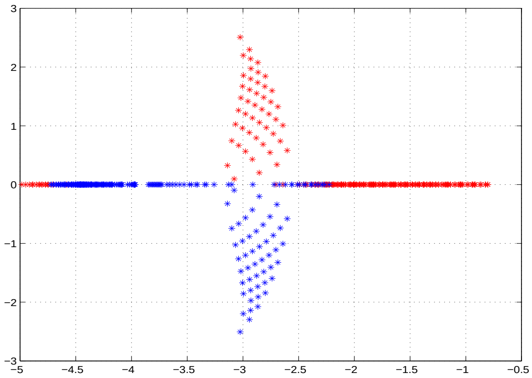

Fig. 1. Finite eigenvalues location of the uncertain closed loop system over the considered polytope

According to Theorem 3, the robust state feedback gain

$$K = \begin{bmatrix} 39.4509 & -16.8080 & 16.9447 \\ -75.3237 & 15.2275 & -59.6286 \end{bmatrix}$$

will renders the uncertain closed loop system robustly admissible.

7 Conclusion

The problem of admissibility and admissibilization for continuous time descriptor systems have been studied. In terms of a strict LMI, a necessary and sufficient condition for continuous descriptor systems to be admissible has been proposed. This condition is an LMI twin formulation of the well known improved generalized Lyapunov equation. LMI conditions are obtained to ensure the admissibility of the closed loop via a state feedback control law. A robust admissible state feedback control law is proposed for polytopic uncertain continuous descriptor systems. Numerical examples are given to illustrate the usefulness of the proposed methods.

References

1. B.-R. Barmish. Necessary and sufficient conditions for quadratic stabilizability of an uncertain system. *J. Optim. Theory Applicat.*, 46:399408, 1985.
2. P. Bernhard. On singular implicit linear dynamical systems. *SIAM Journal of Control Optimization*, 20:612–633, 1982.
3. J. Daafouz and J. Bernussou. Parameter dependent Lyapunov functions for discrete-time systems with time-varying parametric uncertainties. *Syst. Control Lett.*, 43:355359, 2001.

4. L. Dai. *Singular control systems.* Lecture notes in control and information sciences. Springer Verlag, Berlin, 1989.
5. P. Gahinet, P. Apkarian, and M. Chilali. Affine parameter-dependent Lyapunov functions and real parametric uncertainty. *IEEE Trans. Automat. Contr.*, 41:436442, 1996.
6. J.-Y. Ishihara and M.-H. Terra. On the Lyapunov theorem for singular systems. *IEEE Trans. on Automatic Control*, 47:1026–1930, 2002.
7. C. H. Kuo and C. H. Fang. An LMI approach to admissibilization of unceratin descriptor systems via static output feedback. *Proceedings of the American Control Conference*, FP06, 2003.
8. F. L. Lewis. A survey of linear singular systems. *Circuits systems Sig. Prog.*, 5:266–36, 1986.
9. I. Masubuchi and Y. Kamitane. H_∞ control for descriptor systems: A matrix inequality approach. *Automatica*, 33:669–673, 1997.
10. R.-E. Skelton, T. Iwasaki K., and Grigoriadis. *A Unified Algebraic Approach to Linear Control Design.* Taylor-Francis, Bristol-USA, 1998.
11. K. Takaba, N. Morihira, and T. A. Katayama. Generalized Lyapunov theorem for descriptor system. *Systems and Control Letters*, 24:49–51, 1995.
12. A. Varga. On stabilization methods of descriptor systems. *Systems & Control Letters*, 24:133–138, 1995.
13. S. Xu and J. Lam. Robust stability and stabilization of discrete singular systems : An equivalent characterization. *IEEE Trans. on Automatic Control*, 49:568–573, 2004.
14. K. Zhou, J.-C. Doyle, and K. Glover. *Robust and Optimal Control.* Prentice-Hall, Upper Saddle River, NJ, 1996.
15. K. Zhou and P.-P. Khargonekar. Robust stabilization of linear systems with norm-bounded time-varying uncertainty. *Sys. Control Lett.*, 10:1720, 1988.